# Fixed and Random Factors in Mixed Models

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# The Goal Today:

Clear up confusion about vocabulary, concepts, and interpretation of fixed and random factors

# First, some vocabulary and concepts

# What Kind of Models Are we Talking About Here?

- Mixed Models

- Multilevel Models

- Random Effects Models

- Mixed Effects Models

- Hierarchical Linear Models

Extend linear models to include random factors

# The Two Types of Predictor Variables based on: How You Measure Them

**Covariates** — Numerical Predictor Variables

**Factors** — Categorical Predictor Variables

$$Y_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_j$$

# There are Two Types of Covariates

| Independent Variables | Control Variables |
|---|---|
| Key Covariates about whose specific measured values you have hypotheses | Covariates whose specific measured values you need to control for to test your hypothesis |

$$Y_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_j$$

# There are Three Types of Factors

| Independent Variables | Control Variables | Blocking Variables |
|---|---|---|
| Key Factors about whose specific measured values you have hypotheses | Factors whose specific measured values you need to control for to test your hypothesis | Factors you need to control for, but whose measured values could be swapped out for other values and you'd still test the same hypothesis |

$$Y_j = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon_j$$

# Examples of Blocking Variables

**Blocks**

Blocking Variable

**Crop Variety**

Independent Variable

| | Farm 1 | Farm 2 | Farm 3 |
|---|---|---|---|
| Variety 1 | **50 Plants** | **50 Plants** | **50 Plants** |
| Variety 2 | **50 Plants** | **50 Plants** | **50 Plants** |
| Variety 3 | **50 Plants** | **50 Plants** | **50 Plants** |

Soil Nitrogen: Control Variable

Examples of Blocking Variables

# Examples of Blocking Variables

Blocking
Variable

**Types of
Sentence**

**Subject**

Independent
Variable

| | Subject 1 | Subject 2 | ...Subject 30 |
|---|---|---|---|
| Active | | | |
| Passive | | | |
| Reflexive | | | |

THE ANALYSIS
F A C T O R

# Types of Predictors

| Independent Variables | Control Variables | Blocking Variables |
|---|---|---|
| Key Factors about whose specific measured values you have hypotheses | Factors whose specific measured values you need to control for to test your hypothesis | Factors you need to control for, but whose measured values could be swapped out for other values and you'd still test the same hypothesis |
| **Always Fixed** | **Always Fixed** | **Fixed or Random** |

# How We Measure the Effects of Fixed and Random Factors

## Blocks as Fixed

$$Y_j = \beta_0 + \beta_1 IV + \beta_2 Block_1 + \beta_3 Block_2 + \beta_4 Block_3 + \beta_5 Block_4 + \varepsilon_j$$

$\varepsilon_j \sim iid\ N(0, \sigma^2)$
for individual j

Fixed *Effect*

Fixed *Factor*

## Blocks as Random

$$Y_{ij} = \beta_0 + \beta_1 IV + u_i + \varepsilon_{ij}$$

$\varepsilon_{ij} \sim iid\ N(0, \sigma^2)$
for individual j in block i

$u_i \sim iid\ N(0, \tau^2)$
for block i

Random *Effect*

Random *Factor*

Random Intercept Variance

# Fixed vs. Random Factors

|  | Fixed Factors | Random Factors |
|---|---|---|
| Effect measured by: | differences in means | the variance of the distribution of means |
| Results: | limited to the values you chose | generalize to the distribution |
| Number of Parameters: | k-1 | 1 |

# Situations that Indicate Fixed Blocks

1. Too few values to treat as random

2. A specific interest in comparing these values

# Situations that Indicate Random Blocks

1. Too many values to account for as fixed

2. Want to generalize conclusions to other values of blocks

3. Interest in quantifying % overall variation

4. Interest in accounting for variation in this factor

# A More Complicated Model

DV:

Yield

IVs:

Fertilizer: Organic, nitrogen
Beet Variety: 1, 2, or 3

Blocks:

Farm: Farm 1, 2, or 3
Field: 6 Fields

| Organic | Nitrogen |
|---|---|
| Variety 2 | Variety 3 |
| Variety 3 | Variety 1 |
| Variety 1 | Variety 2 |

# A More Complicated Model

Example 2

### DV:

Math Scores

### IVs:

Program: 3 math programs

### Blocks:

Students: 18-24 students per class
School: 12 schools
Class: 2-4 classes per school

### Controls:

Teacher's highest degree earned
Child sex: male or female

### Covariates:

School % free lunch

Teacher's years of experience

Student's previous year math score

# A More Complicated Model

DV:

Reading time

Covariates:

Age

IVs:

Language status: Native, Advanced, Intermediate
Sentence Type: actional, psychological, descriptive

Blocks:

Sentence: 10 of each type
Subject: 30 participants each

# References and Further Reading

West, Welch, Galecki. Linear Mixed Models
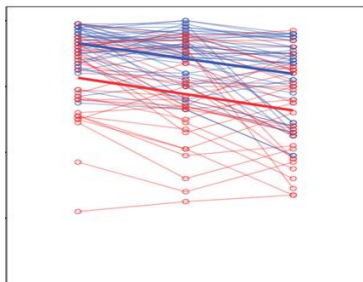
Snijders & Bosker. Multilevel Analysis

Douglas Montgomery. Design and Analysis of Experiments

Resources on our site: http://www.theanalysisfactor.com/resources/by-topic/mixed-multilevel-models/

Random Intercept and Random Slope Models
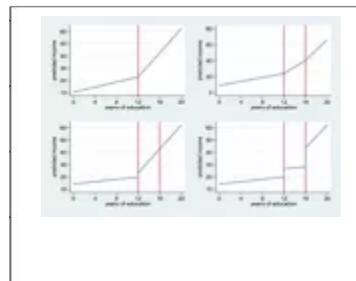
# If you want to learn more: Online Workshops



**Analyzing Repeated Measures Data: ANOVA and Mixed Model Approaches**

**Instructor:** Karen Grace-Martin

**Stage:** 3, Beyond Linear Models

**Software:** SPSS, SAS, R, Stata



**Linear Models**

**Instructor:** Jeff Meyer

**Stage:** 2, Master Linear Models

**Software:** SPSS, SAS, R, Stata

https://www.theanalysisfactor.com/live-online-workshops/

# Questions?